Students analyze the characteristics of sustainably designed items. In a real-world application exercise, students calculate the surface area and volume of a prism and a cylinder in order to connect geometric properties to sustainable design principles. A closing discussion raises questions about the applicability of sustainable design to different situations, as well as its social and economic impacts.
Critical Thinking Questions

• How is the design of an object tied to creating a well-balanced environment, society, and economy?
• What are objects you could envision being redesigned?

Objectives

• Calculate surface area and volume of cylinders and prisms
• Use geometric measurements to improve the sustainability of product design
• Investigate resource use as one consideration of sustainable design

Key Concepts

• Surface area of prism and cylinder
• Volume of prism and cylinder
• Sustainable design
• Fuel efficiency

NCTM Standards and Expectations Addressed

Geometry: Precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties

Geometry: Use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume

Geometry: Recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life

Measurement: Select and apply techniques and tools to accurately find length, area, volume, and angle measures to appropriate levels of precision

Problem Solving: Solve problems that arise in mathematics and in other contexts

Reasoning and Proof: Make and investigate mathematical conjectures

Communication: Communicate mathematical thinking coherently and clearly to peers, teachers, and others

Connections: Recognize and apply mathematics in contexts outside of mathematics

Representation: Use representations to model and interpret physical, social, and mathematical phenomena

Materials/Preparation

Overhead: Milk Jug Designs
6 - 1 gallon milk jugs
6 crates or boxes of equal size that could fit at least 4 milk jugs (such as a paper box)
Handout: Taking Shape, 1 per student
Teacher master: Taking Shape

Background and Practice

Student reading: Designer Products
Practice worksheet: Practice with Surface Area & Volume
**Teacher Instructions**

1. Ask students what kinds of things they use every day that someone has designed. *There are infinite answers here—backpacks, cups, pens, cars, etc.*

2. Challenge students to think of how the design of a particular item or building suits its use. *For example, a sturdy shopping bag with a reinforced bottom is suitable for heavy objects. A building with south-facing windows is ideal for receiving natural light. Can they think of examples of poorly designed objects? For example, a lotion container that makes it difficult to get the lotion out of the container, or a small bathroom stall where the door opens inward.

3. Ask students to name a 3-dimensional geometric figure that resembles a plastic milk jug. *Some might say it resembles a cylinder or a prism.*

4. Split students into groups of 3-4 students.

5. Hand out a milk jug and a crate or box to each group.

6. Read the following scenario to them: *“Your group is part of a milk distribution company. You are trying to figure out how to make the transport of milk from dairy farms to stores across the nation as cost-efficient and environmentally-responsible as possible. Your first task is to determine how many 1-gallon milk jugs can fit into the crate you were given.”*

7. Give groups 1 minute to estimate how many of their milk jugs could fit into their crate/box.

8. Ask students if there would be any unused space in the crate/box if it was filled with milk jugs. What factors contribute to this wasted space? *Jugs cannot be stacked on top of each other; the handle seems to be wasted space; the neck is narrower than the body of the jug.*

9. Ask students how the use of milk jugs and crates/boxes like these is related to the cost-effectiveness of transporting milk from a dairy farm to a store. *Because of wasted space, more trips might have to be made; small crates mean that more crates need to be used.*

10. Give groups 1 final minute to brainstorm how the milk jug could be redesigned so that more could fit onto a truck.

11. Ask a representative from each group to share his or her group’s ideas.

12. Explain to the class that they just did an exercise in **sustainable design**. *Sustainable design refers to designing products and buildings with sustainability in mind—that is, maximizing economic benefits while minimizing negative impacts on people and environmental resources. Sustainable design can be used to design almost anything, including milk jugs!*
13. Pass out a *Taking Shape* handout to each student. You may want to allow students to work together in pairs. If they are unsure of the formulas for calculating volume and surface area, share the following formulas with them:

- **cylinder volume:**
  \[ V = \pi r^2 h \]

- **prism volume:**
  \[ V = Bh, \text{ where } B \text{ is the area of the base} \]

- **cylinder surface area:**
  \[ SA = 2\pi r^2 + 2\pi rh \]

- **prism surface area:**
  \[ SA = 2(\text{area of base}) + 2(\text{area of front side}) + 2(\text{area of right side}) \]

14. Note that for several of the questions on the handout (1, 5, 8, 9, and 10), there will likely be a variety of answers. The objective with these questions is for students to think critically about the link between geometry and sustainable design.

15. After students complete the worksheet, lead a class discussion using the following questions.

**Discussion Questions**

1. In 2008 Sam’s Club began to use a new milk jug design, shaped somewhat like a rectangular prism, to cut down on shipping costs. More of the new milk jugs can fit in a single truck, resulting in fewer trips. Not everyone was happy about the switch—customers complained that it is difficult to pour the milk without the handle and narrow neck that the old jugs had. How could Sam’s Club help people adjust to using the new containers?

2. Why do you think sustainable design has to consider the economic and social effects of new designs, rather than just the environmental impacts?

3. Would you buy sustainably designed items even if they looked very different than what you are used to? Why or why not?

4. Would you pay more for products that were designed with sustainability in mind?
Extension Ideas

1. Sam’s Club sells gallons of milk in plastic containers similar to the cylinder-based design. They claim that it has dropped the cost of a gallon of milk from $2.58 to $2.18. Calculate the percent savings to the customer for a gallon of milk sold in the new containers.¹

2. If the new containers sold by Sam’s Club contain 4.5 gallons of milk for every 1 ft³ of truck space, which is 50% more than the old containers,² how many gallons of milk per cubic foot did the old jugs hold?

3. How many of the new milk jugs (13 cm x 13 cm x 25 cm) will fit in a truck whose trailer is 28 feet long, 10 feet high, and 10 feet wide?

*Hint: The measurements given for the jugs are in different units than the truck measurements. In order to work with the same units, you will first need to convert ft to cm, or cm to ft.*

4. Can you design an even more efficient container than the prism and cylinder designs? Using clay, try designing containers based on other polyhedron shapes, such as pyramids or cones. The objective is to reduce wasted space on trucks that ship the milk from dairies to grocery stores.

Additional Resources

- [www.usgbc.org](http://www.usgbc.org)—The U.S. Green Building Council provides information about green building certification (the Leadership in Energy and Environmental Design rating system, or LEED). Their website includes an Educator Resource Center.

Action Project

Have students research products available in their community that they consider sustainable or sustainably designed. Put together a guide alerting citizens to where they can find these products. Restaurants and grocers that sell sustainably produced food (such as Fair Trade coffee or organically grown produce) could also be included in the guide. Distribute the guide at local shops and restaurants that sell these products.
Milk Jug Designs

Traditional Milk Jug

Redesigned Milk Jug
Investigations

1. We are going to examine 2 milk container designs to see which one is more sustainable. But first, write down at least 3 ways that the concept of sustainable design could be used to redesign plastic milk jugs. **Sustainable design** involves making products in a way that minimizes negative impacts of production on people and environmental resources while still balancing economic costs.

2. Calculate volume for the following 2 containers:

<table>
<thead>
<tr>
<th>Container A</th>
<th>Container B</th>
</tr>
</thead>
<tbody>
<tr>
<td>25 cm x 13 cm</td>
<td>25 cm x 13 cm</td>
</tr>
</tbody>
</table>

3. Which container holds a greater volume of milk?

4. What feature(s) of this container allows it to hold more milk?

5. How is a container’s volume related to sustainable design?

6. Now calculate and compare the surface area of the 2 containers. Round answers to 2 decimal places (the hundredths place).

7. Which container has a greater surface area?

8. How is surface area related to sustainable design?

9. Of the 2 containers, which one would you recommend stores carry? Take into consideration volume, surface area, production and transport costs, the environment, and usability.

10. What other design features would you add to the preferred container to make it even easier to use? Explain how these features would affect the sustainability of the container’s design, either positively or negatively.
ANSWERS

1. Just a few possibilities include:
   - reducing volume so that containers take up less space in store refrigerators (saving energy) and less space in delivery trucks (reducing fuel use)
   - reducing surface area so that less packaging is required (saving resources)
   - using recycled plastic
   - making sure the container can be easily recycled or reused

2. Container A
   \[ V = \pi r^2 h \]
   \[ V = \pi (6.5)^2 (25) = 3318.3 \text{ cm}^3 \]

   Container B
   \[ V = Bh \]
   \[ V = (13)(13)(25) = 4225 \text{ cm}^3 \]

3. Container B

4. Container B has a larger base area, resulting in greater volume.

5. A container with more volume holds more milk, reducing the number of containers (and often the amount of packaging) used.

6. Container A
   \[ SA = 2\pi r^2 + 2\pi rh \]
   \[ = 2\pi (6.5)^2 + 2\pi (6.5)25 \]
   \[ = 265.46 + 1021.02 \]
   \[ = 1286.48 \text{ cm}^2 \]

   Container B
   \[ SA = 2(13 \cdot 13) + 2(25 \cdot 13) + 2(25 \cdot 13) \]
   \[ = 338 + 650 + 650 \]
   \[ = 1638 \text{ cm}^2 \]

7. Container B

8. A large surface area requires more packaging materials (plastic, in this case). Items with a small surface area could be considered more sustainable.

9. Possible answer: Stores should carry Container B. Even though its surface area is larger, it has a greater volume, and the containers can be stacked on top of each other. This means more containers can fit into a truck. Less fuel would be used to transport the containers because delivery trucks wouldn’t have to make as many trips.

10. Features such as handles and a narrow neck for pouring would make the container easier to use, but would likely diminish the environmental and economic sustainability of the container. Another idea is to fashion the container from material that insulates the milk better so that the refrigerator doesn’t need to be kept as cold. This would improve the sustainability of the container. The sky is the limit with sustainable design ideas!
Surface area is the sum of the areas of the faces or surfaces of a 3-dimensional object.

The area of any side of the above cube is 4 in². 
(2 in × 2 in = 4 in²)

In the case of a cube, all 6 sides have the same area.

The surface area can be found by adding together the areas of all sides:
4 in² + 4 in² + 4 in² + 4 in² + 4 in² + 4 in² = 24 in²

Volume is the number of cubic units (such as cm³) that are needed to fill a 3-dimensional figure.

For a cube or prism, the volume (V) is the base area (B) multiplied by the height (h).

In this example, B = 2 in × 2 in = 4 in².
V = Bh = 4 in² × 2 in = 8 in³

That means that 8 cubic inches can fit inside the cube.

1. Juice cartons can be recycled into notebook covers. Calculate the surface area of the following juice carton in square centimeters (cm²).

2. A company wants to make small notebooks out of juice cartons. The notebooks have front and back covers that are each 15 cm long and 10 cm wide. How many complete notebooks can be made from a single juice carton?

3. A different company wants to devise a way to recycle the juice cartons, sanitize them, and refill them. What is the maximum volume (in cubic centimeters, or cm³) of liquid that each carton can hold?

ANSWERS
1. SA = 2(90.25) + 4(180.5) = 902.5 cm²
2. A = 15 × 10 = 150 cm²
   902.5 cm² ÷ 150 cm² ≈ 6.02
   = 6 notebooks
3. V = 9.5 cm × 9.5 cm × 19 cm = 1714.75 cm³